computational simulation that is to match experimental data. Inclusion of the experimental normal velocity was shown to be vital for the derivation of correct computational boundary conditions. Streamwise velocity profiles determined from pitot probe measurements were found to be insufficient for this purpose; hence, techniques capable of determining both streamwise and normal velocity must be employed.

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Scaling of Incipient Separation in Supersonic/Transonic Speed Laminar Flows

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Nomenclature

C	= Chapman-Rubesin parameter, $\mu T_\infty/\mu_\infty T$
L	= reference length (see Fig. 1)
M	= Mach number
p	= static pressure
Re_L	= Reynolds number $\cong \varepsilon^{-8}$, $\rho_\infty U_\infty L/\mu_\infty$
T	= absolute static temperature
U_∞	= freestream velocity at edge of incoming boundary layer
u, v	= velocity components in x, y directions, respectively
x, y	= streamwise and normal coordinates, respectively
β	= $(M_\infty^2 - 1)^{\frac{1}{2}}$
δ^*	= displacement thickness variable
θ	= flow deflection angle
μ	= coefficient of viscosity, $\equiv \rho\nu$
ρ	= density
χ	= viscous interaction parameter, $M_\infty^3 Re_L^{-\frac{1}{2}}$
ω	= viscosity temperature-dependence exponent ($\mu \sim T^\omega$)

Subscripts

ADIAB	= adiabatic wall conditions
$i.s.$	= incipient separation
REF	= based on reference temperature
w	= wall surface conditions

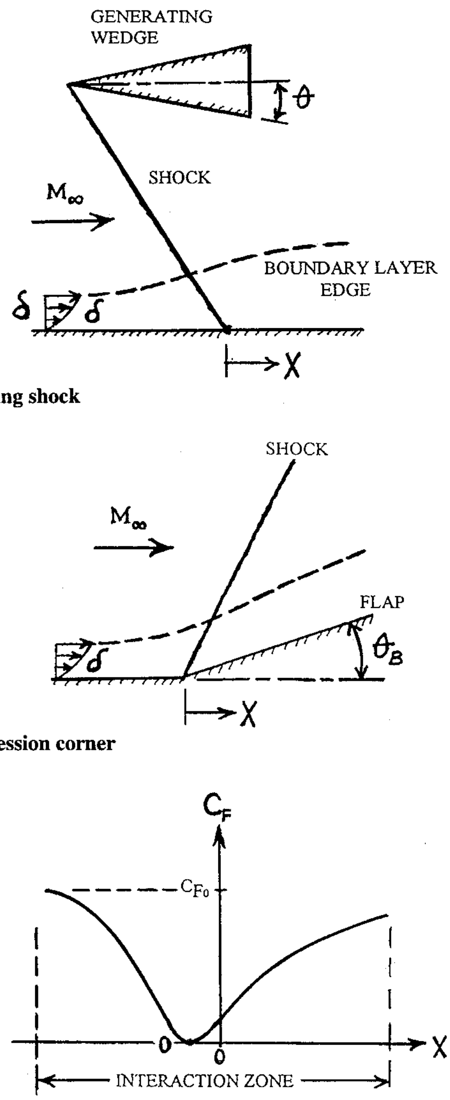
Superscript

$(\hat{\ })$	= nondimensional variables from triple-deck theory

Introduction

INTERACTIONS between oblique shock waves and boundary layers must be understood to predict the performance of aerodynamic devices such as flaps, spoilers, and inlets. These involve strong viscous/inviscid interaction flow with a large local adverse pressure gradient that often provokes boundary-layer separation. The prediction of the onset of such separation and the delineation of the underlying scaling laws that govern it continue to be important in aerodynamic studies of high speed aircraft and missiles, these vehicles operate and are tested over a wide range of Mach and Reynolds numbers. This paper re-examines the fundamental similitude rules pertaining to the laminar (high-altitude) flight regime of supersonic vehicles, with the goal of establishing a single unified scaling law for both supersonic and moderately hypersonic Mach numbers.

We consider two-dimensional steady laminar flow of an ideal gas that undergoes incipient separation in the strong interaction region associated with either an impinging shock or a compression corner (Fig. 1). This event is characterized by a critical value of the forcing function (e.g., a deflection angle or nondimensional shock pressure



a) Impinging shock

b) Compression corner

c) Definition of incipient separation

Fig. 1 Regions of local shock/boundary layer interaction.

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ratio) as a function of M_∞, Re_L, and the wall temperature ratio $T_w/T_{W,\mathrm{ADIAB}}$. Now, an oft-cited criterion for such separations in high-speed flows is[1]

$$M_\infty \theta_{i.s.} \approx \ell \chi^{\frac{1}{2}} \tag{1}$$

where the constant $\ell \approx 70$–80 depends on T_w. Although Eq. (1) is only semi-empirical,[2] it has withstood the test of time in successfully correlating a substantial body of both experimental data[3,4] and numerical Navier-Stokes solutions.[1] The present work seeks a theoretical foundation for Eq. 1 and a twofold extension of it that applies to transonic as well as supersonic Mach numbers and also captures the nonadiabatic wall temperature effect on the constant ℓ.

The method of approach is the so-called triple-deck theory in its leading high-Reynolds-number approximation, recast into a form suited to nonadiabatic flows by means of the reference temperature concept. To fix ideas, we specifically treat compression corner interactions (Fig. 1b); the companion impinging shock problem follows similar lines.

Formulation of the Analysis

It is known that the disturbance flow physics within the short-ranged viscous-inviscid interaction zone organizes itself into three distinct "decks" when the Reynolds number is high[5]: an outer layer external to the boundary layer consisting of potential disturbance flow associated with the viscous displacement effect of the underlying decks, a middle layer within the incoming boundary-layer thickness containing rotational inviscid disturbance flow dependent on the boundary-layer profile, and a thin inner deck of viscous-disturbance flow within the linear portion of this velocity profile that is interactively coupled with the local pressure field. In terms of the basic small perturbation parameter $\varepsilon \equiv Re_L^{-\frac{1}{8}}$, these decks have thicknesses of order $\varepsilon^3 L$, $\varepsilon^4 L$, and $\varepsilon^5 L$, respectively, along a streamwise interaction zone length of order $\varepsilon^3 L$. Taken in its leading asymptotic approximation as $\varepsilon \to 0$ (very high Reynolds number), this triple-deck theory yields the following set of governing disturbance flow equations when the external Mach number ranges from supersonic to moderately hypersonic[5,6]:

$$\frac{v}{u} \cong \frac{v_e}{U_\infty} \cong \frac{\mathrm{d}}{\mathrm{d}x}(y_B + \delta^*) \qquad \text{[middle deck]} \tag{2}$$

$$\left.\begin{aligned} &\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ &u\frac{\partial u}{\partial u} + v\frac{\partial u}{\partial y} + \rho_w^{-1}\frac{\mathrm{d}p}{\mathrm{d}x} = \nu_w \frac{\partial^2 u}{\partial y^2} \end{aligned}\right\} \qquad \text{[inner deck]} \tag{3}$$

with the no-slip condition $u(x, y_B) = v(x, y_B) = 0$ on the impermeable body surface $y_B = \theta_B x$ at $x > 0$. Furthermore, the inner-middle deck matching condition yields the relationship

$$u(x, y \to \infty) = \frac{\tau_{w_0}}{\mu w_0}(y - y_B - \delta^*) \tag{4}$$

The variable δ^* here is an unknown displacement thickness that is linked to the interactive pressure perturbation as shown later. The wall shear τ_{w_0} here will be evaluated at a suitably defined reference condition. According to the leading asymptotic approximation, Eqs. (2–4) are valid for all values of M_∞ and $T_W/T_{W,\mathrm{ADIAB}}$ including the presence of wall heat transfer.

The foregoing quartet of equations must be supplemented by a further relationship between the interactive pressure and displacement thickness δ^*, and it is here that the analysis becomes specialized to the Mach number regime and streamwise scale adopted for study. In the present work we are concerned exclusively with the local short-scale events in the immediate vicinity of the compression corner to the exclusion of any larger scale effects such as a blunt nose or leading edge upstream on the body. For supersonic to moderately hypersonic flow, we adopt the p_e vs v_e/u_e relationship[7,8]

$$\frac{p - p_\infty}{\gamma p_\infty M_\infty^2} \approx v_e/\beta U_\infty \tag{5}$$

We now formulate this triple-deck model using the reference temperature method. According to this method[6] the combined effects of compressibility and heat transfer on the incoming boundary layer can be accurately accounted for by evaluating its physical properties at a reference temperature T_{REF} that depends on γ, M_∞, and $T_W/T_{W,\mathrm{ADIAB}}$. Adopting Eckerts result (which has been shown[7] to have a fundamental basis in compressible boundary-layer theory), this is for air

$$\frac{T_{\mathrm{REF}}}{T_\infty} \cong 0.50 + 0.039 M_\infty^2 + 0.50\frac{T_w}{T_\infty} \tag{6}$$

We further take $\mu \sim T^\omega$ with $0.50 \le \omega \le 1$. Then introduce the nondimensional rescaled variables[5]

$$\hat{x} \equiv [(x-L)/L]\lambda^{\frac{5}{4}}\beta^{\frac{3}{4}}/\varepsilon^3 C_{\mathrm{REF}}^{\frac{3}{8}}(T_{\mathrm{REF}}/T_\infty)^{\frac{3}{2}}(T_w/T_{\mathrm{REF}})^{\omega+\frac{1}{2}} \tag{7}$$

$$\hat{y} \equiv (y/L)\lambda^{\frac{3}{4}}\beta^{\frac{1}{4}}/\varepsilon^5 C_{\mathrm{REF}}^{\frac{5}{8}}(T_{\mathrm{REF}}/T_\infty)^{\frac{3}{2}}(T_w/T_{\mathrm{REF}})^{\omega+\frac{1}{2}} \tag{8}$$

$$\hat{u} \equiv (u/U_\infty)\beta^{\frac{1}{4}}/\varepsilon C_{\mathrm{REF}}^{\frac{1}{8}}\lambda^{\frac{1}{4}}(T_w/T_\infty)^{\frac{1}{2}} \tag{9}$$

$$\hat{v} \equiv (v/U_\infty)/\varepsilon^3\beta^{\frac{1}{4}} C_{\mathrm{REF}}^{\frac{3}{8}}\lambda^{\frac{3}{4}}(T_w/T_\infty)^{\frac{1}{2}} \tag{10}$$

$$\hat{p} \equiv \left[(p - p_\infty)/\rho_\infty U_\infty^2\right]\beta^{\frac{1}{2}}/\varepsilon^2 C_{\mathrm{REF}}^{\frac{1}{4}}\lambda^{\frac{1}{2}} \tag{11}$$

$$\hat{\theta} \equiv \theta/\varepsilon^2\lambda^{\frac{1}{2}}\beta^{\frac{1}{2}} C_{\mathrm{REF}}^{\frac{1}{4}} \tag{12}$$

$$\hat{\delta}^* \equiv (\delta^*/L)\lambda^{\frac{3}{4}}\beta^{\frac{1}{4}}/\varepsilon^5 C_{\mathrm{REF}}^{\frac{3}{8}}(T_{\mathrm{REF}}/T_\infty)^{\frac{3}{2}}(T_w/T_{\mathrm{REF}})^{\omega+\frac{1}{2}} \tag{13}$$

where $\lambda = 0.332$ and $C_{\mathrm{REF}} \equiv \mu_{\mathrm{REF}} T_\infty / T_{\mathrm{REF}}\mu_\infty$ are the Blasius and Chapman-Rubesin constants, respectively. One then finds that Eqs. (2–5) yield the following well-known universal equations:

$$\hat{p} = \hat{v}_e \qquad \text{[outer deck]} \tag{14}$$

$$\hat{v} \cong \hat{v}_e = \frac{\mathrm{d}}{\mathrm{d}\hat{x}}(\hat{y}_B + \hat{\delta}^*) \qquad \text{[middle deck]} \tag{15}$$

$$\left.\begin{aligned} &\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0 \\ &\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} + \frac{\mathrm{d}\hat{p}}{\mathrm{d}\hat{x}} = \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} \end{aligned}\right\} \qquad \text{[inner deck]} \tag{16}$$

$$\hat{u}(\hat{x}, \hat{y} \to \infty) = \hat{y} - \hat{y}_B - \hat{\delta}^* \qquad \text{[middle-inner deck matching]} \tag{17}$$

subject to the boundary conditions $\hat{u} = \hat{v} = 0$ along $\hat{y} = 0$ for $\hat{x} < 0$ and $\hat{y} = \hat{y}_B = \hat{\theta}_B \hat{x}$ for $\hat{x} \ge 0$.

Incipient Separation Criteria

The analytical and numerical solution of Eqs. (14–17) has been studied extensively.[5,8,9] These studies show that the local wall shear stress vanishes when the nondimensional pressure takes the universal value

$$\hat{p}_{i.s.} \cong 1.03 \tag{18}$$

We note that since pressure is necessarily proportional to the wall deflection angle, we can write $\hat{\theta}_{i.s.} = K_s \hat{p}_{i.s.}$ where $K_s \cong 1.52$ (Ref. 8). The desired scaling law for $M_\infty \theta_{i.s.}$ may now be obtained by reversing the transformation, Eq. (12), which yields

$$M_\infty \theta_{i.s.} = K_s \lambda^{\frac{1}{2}} \left[\left(\frac{\sqrt{M_\infty^2 - 1}}{M_\infty}\right) C_{\mathrm{REF}}^{\frac{1}{2}} \chi\right]^{\frac{1}{2}} \tag{19}$$

In the hypersonic limit $M_\infty^2 \gg 1$, this passes over directly to the form of Eq. (1), thereby establishing the latter within the framework of triple-deck theory. Moreover, Eq. (19) extends Eq. (1) to lower supersonic Mach numbers; this consists in multiplying the interac-

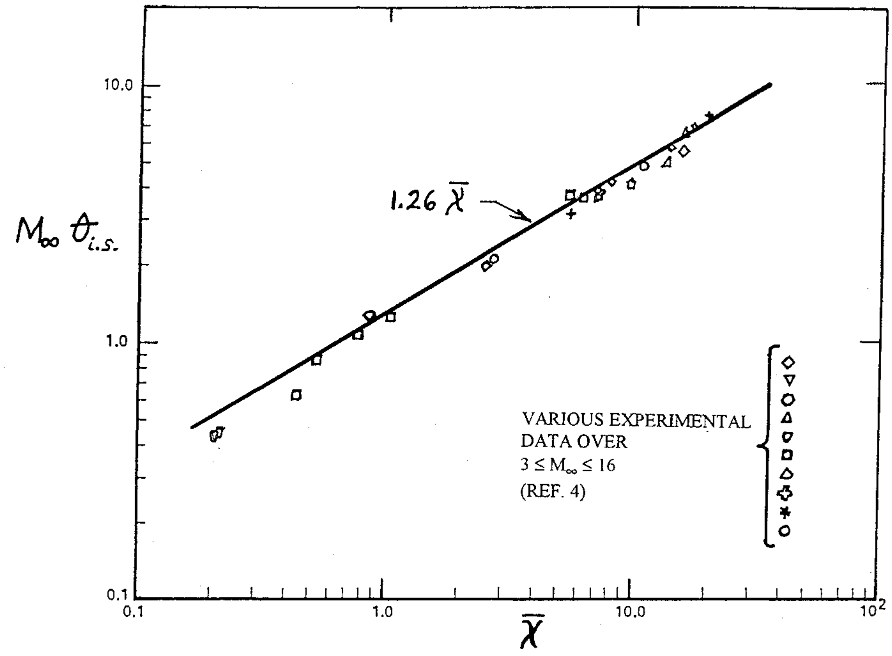

Fig. 2 Correlation of experimental data.

tion parameter χ in Eq. (1) by the correction factor β/M_∞. A typical successful correlation by Eq. (19) of experimental data over a wide range of conditions is illustrated in Fig. 2. We note that this equation further implies $\theta_{i.s.} \approx [(M_\infty^2-1)/Re_L]^{\frac{1}{4}}$ i.e., a slow reduction of $\theta_{i.s.}$ with either increasing Re_L or decreasing M_∞. As regards the wall temperature effect, it is contained entirely in the parameter C_{REF} to the present approximation (see the following text).

One may alternatively express the foregoing incipient separation criterion in terms of pressure rise. Thus converting Eq. (18) back to physical variables yields

$$\frac{p_{i.s.}-p_\infty}{\gamma p_\infty} \sim \left[\left(\frac{M_\infty}{\sqrt{M_\infty^2-1}}\right)C_{\text{REF}}^{\frac{1}{4}}\chi\right]^{\frac{1}{2}}$$

$$\sim M_\infty^2\left(C_{\text{REF}}/Re_L\sqrt{M_\infty^2-1}\right)^{\frac{1}{4}} \tag{20}$$

The latter relation is, in fact, a well-known scaling law for the free interaction zone of both ramp- and impinging shock-generated laminar separating flows at supersonic speeds.[10]

Wall Temperature Effect

In the present leading asymptotic approximation, this effect is contained within the parameter C_{REF}, evaluated from Eq. (6). To develop the consequences of this in a more revealing engineering form, we write

$$C_{\text{REF}}^{\frac{1}{4}} = \left(\frac{T_{\text{REF}}}{T_\infty}\right)^{\frac{\omega-1}{4}} = \left(0.5+0.5\frac{T_w}{T_\infty}+0.039M_\infty^2\right)^{\frac{\omega-1}{4}} \tag{21}$$

Since $1.2T_{W,\text{ADIAB}}/T_\infty \cong 1+0.2M_\infty^2$ for air, we can rewrite this as

$$C_{\text{REF}}^{\frac{1}{4}} = \left[0.039\left(1+\frac{5}{M_\infty^2}\right)\right]^{\frac{\omega-1}{4}} M_\infty^{(\omega-1)/2}$$

$$\times\left(1+\frac{1.30}{T_{W,\text{ADIAB}}/T_\infty}+\frac{2.13T_w}{T_{W,\text{ADIAB}}}\right)^{\frac{\omega-1}{4}} \tag{22}$$

In the hypersonic case where $M_\infty^2 \gg 1$ and noting that $(\omega-1)/4$ is small compared with unity, expansion of the last term of Eq. (22) for highly cooled walls yields

$$C_{\text{REF}}^{\frac{1}{4}} \cong [0.466+0.534\omega]\left(0.039M_\infty^2\right)^{\frac{\omega-1}{4}}$$

$$\times\left[1-\frac{(1-\omega)}{(0.88+\omega)}\left(\frac{T_w}{T_{W,\text{ADIAB}}}-1\right)\right] \tag{23}$$

involving the product of an adiabatic wall compressibility effect factor proportional to $M_\infty^{(\omega-1)/2}$ and a second factor in square brackets that represents the relative effect of heat transfer.

The nonadiabatic wall effect predicted by Eqs. (19) and (23) can be compared with a data correlation given by Needham[11] for highly cooled hypersonic interactive separations; for the present T_{REF} model, his results can be shown to take the form

$$\theta_{i.s.} \cong (\theta_{i.s.})_{\text{ADIAB}}\left[1-\left(\frac{1-\omega+0.624}{0.624+\omega}\right)\left(\frac{T_w}{T_{W,\text{ADIAB}}}-1\right)\right] \tag{24}$$

Comparison shows that the leading asymptotic approximation for $\omega = 0.50$ captures 60% of the observed total heat transfer effect via its influence on C_{REF} from a nonlinear $\mu(T)$ relationship (such influence vanishes for the value $\omega = 1$ associated with Stewartsons' "model fluid" approach[12]). Furthermore it is seen that the correct Mach number dependence of the interaction parameter for these separation flows is not M_∞^3 but actually $M_\infty^{2+\omega}$ for an adiabatic wall, in agreement with an unpublished investigation by Cheng.[13]

Extention to $M_\infty > 1$ Transonic Flow

Although perhaps of only academic interest, the foregoing theory can be extended down to transonic (albeit supersonic) Mach numbers in the outer deck by an appropriate modification of its pressure/streamline deflection relationship. Thus, if $M_\infty - 1$ is positive but small compared with unity, such purely wavelike disturbance flow is governed by the nonlinear relationship[14]

$$\frac{p-p_\infty}{\gamma p_\infty M_\infty^2} = \frac{\left(M_\infty^2-1\right)}{(\gamma+1)}\left\{1-\left[\frac{3}{2}\frac{(\gamma+1)v_e/U_\infty}{\left(M_\infty^2-1\right)^{\frac{3}{2}}}\right]^{\frac{2}{3}}\right\} \tag{25}$$

It is seen that when (M_∞^2-1) is sufficiently large, this passes over to the usual linearized supersonic flow relationship between p and v_e/u_∞. As for the flow relationship governing the middle and inner deck disturbance fields, in the leading $\varepsilon \to 0$ approximation they remain the same as those given in Eqs. (2–4), including the wall boundary conditions. If we then introduce the same nondimensional variable transformations as before, we again obtain Eqs. (15–17) of the "canonical" triple-deck formulation with only the pressure relationship Eq. (14) generalized to

$$\hat{p} = \frac{K_T}{\gamma + 1}\left\{1 - \left[1 - \frac{3}{2}\frac{(\gamma + 1)}{K_T}\frac{\mathrm{d}}{\mathrm{d}\hat{x}}(\hat{y}_B + \hat{\delta}^*)\right]^{\frac{2}{3}}\right\} \tag{26}$$

where $K_T \equiv (M_\infty^2 - 1)^{\frac{5}{4}}/\varepsilon^2\lambda^{\frac{1}{2}}C_{\mathrm{REF}}^{\frac{1}{4}}$ is a transonic viscous interaction parameter. In the $K_T \gg 1$ limit, Eq. (26) reduces to Eq. (10) for higher Mach number supersonic flow.

Now, numerical studies of this modified transonic triple-deck problem have shown[15] that the value of $\hat{p}_{i.s.}$ changes very little over a wide range of K_T values: for $K_T \geq 1$, we may thus continue to use the incipient separation criterion Eq. (14). Then with $\mathrm{d}\hat{\delta}^*/\mathrm{d}\hat{x} = K_d\hat{\theta}_{i.s.}$, inversion of Eq. (22) followed by conversion back to physical variables yields

$$M_\infty\theta_{i.s.} \approx \frac{2(1 + K_\mathrm{d})^{-1}}{3(\gamma + 1)}M_\infty\beta^3\left\{1 - \left[1 - 1.03(\gamma + 1) \times \left(\frac{\sqrt{M_\infty^2 - 1}}{M_\infty}\chi C_{\mathrm{REF}}^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{\frac{3}{2}}\right\} \tag{27}$$

This is the desired extension of Eq. (19) to include the transonic regime. Although a bit complicated, it reveals the correct underlying scaling behavior while passing over to the simpler form of Eq. (19) for sufficiently small values of χ. In this regard we note that Eq. (27) is, in fact, a unified scaling law for the entire supersonic regime from transonic to moderately hypersonic.

Acknowledgments

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Simple Method of Supersonic Flow Visualization Using Watertable

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Introduction

THE direct shadowgraph technique is a relatively easy and common means for studying hydrodynamic phenomena in the flow of liquids. A number of visualization techniques for water flow currently exist, the use of which depend mainly on the desired test information, available facilities, and model flow speeds. An overview of the available flow visualization methods has been discussed by Werlé.[1] Another common way of visualizing the flow of liquids is to introduce optical disturbances in the liquid and to detect them by the Schlieren System[2] or the recently developed Sugar Schlieren System.[3] Flow visualization for liquid flow may be conducted in virtually any type of horizontal channel utilizing a smooth transparent bed of shallow water over a glass sheet.

Quantitative assessments of the behaviors of a flow from the recorded flow pattern obtained from conventional flow visualization techniques have often posed great difficulty to researchers. Yamamoto et al.[4,5] have developed an image processing technique from the flow pattern obtained in a free-surface watertable using inclined grid Moiré topography. Recently, a relatively simple and novel optical method for the quantitative evaluation of physical flow variables in a high-speed flow has been obtained from the photographs of the flow pattern obtained in a watertable using the established theory of hydraulic analogy.[6] This optical technique for processing the image patterns obtained photographically in a watertable seems to be new for the study of two-dimensional flowfield quantitatively. The present photographic method for flow visualization in a watertable has been demonstrated at supersonic speeds and may also be extended to study different model configurations.

Method and Procedure

The schematic arrangement of the experimental setup used to visualize the flowfield is illustrated in Fig. 1. Great care is necessary during the experiment to rid the flowfield of surging, turbulence, and angularity. A conventional 35-mm camera and black and white film are used to record the flow pattern. The picture of the flowfield obtained using this new technique as the flowfield moved past a symmetric airfoil with a sharp leading edge at a Froude number (corresponds to the Mach number in a gas flow) greater than unity is shown in Fig. 2. The flow is moving from top to bottom about the model. The introduction of the alternate bright and dark bands (marked as mechanical grating plate in Fig. 1) add much greater contrast to the image of the flow pattern which, incidentally, has a strong resemblance to the conventional interferogram. It differs from the conventional interferogram, however, for the resolution of the flowfield depends on the width, spacing, as well as the fineness of the parallel equidistant bands made on a transparent sheet.

The method of visualization involves superimposing the shadows of alternate bands on the flow placed in a plane at right angles to the flow direction. The position of the light sources used to illuminate the flowfield is important to get a better resolution of the image of

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